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Two coupled Luttinger chains

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Abstract. Two Luttinger chains coupled by an attractive interaction are investigated. Because of the attractive interaction the charge-$1e$ excitations have a gap, in contrast to a holon with charge $2e$ which is massless. A superconductor order parameter for the two charges is introduced. The pair–pair correlation is computed. We find a power-law behaviour controlled by the $2e$ Luttinger excitations. We comment on the relevance of this work to the recent suggestions for triplet pairing between planes and s waves in the plane. We mention the possibility of a persistent current with a period of $\Phi_{0/2} = hc/2e$ in coupled rings.

Recently Anderson [1] has suggested that the physics of strongly correlated systems in one and two dimensions are described by the Luttinger liquid. One of the possible consequences of this fact is that superconductivity will arise when two Luttinger liquids are coupled.

In order to investigate this possibility we shall study the effect of an attractive interaction between two metallic chains. Instead of normal metal we shall assume that our chains are described by a Luttinger liquid. One of the important features of the Luttinger liquid is that the fermion excitations are decomposed into holon (charge $1e$) and spinon (spin $\frac{1}{2}$) excitations. These two excitations propagate with different velocities. When we add an attractive interaction between the chains, the interaction induces a gap in the charge-$1e$ holon, giving rise to a charge-$2e$ holon which propagates as an acoustic wave.

Formally we use the bosonization [2] method and decompose the fermions into two bosons: the holon and the spinon. The two chains are described by an isospin index $\tau = +, -$. Therefore each bosonic field has two indices $\sigma = \uparrow, \downarrow$, $\tau = +, -$, spin and isospin. The Hamiltonian which describes this problem is given by

$$H = H^0 + H_{\text{int}}. \tag{1a}$$

H^0 is the Hamiltonian for the two uncoupled Luttinger chains. Since in each Luttinger chain we have spin and charge separation, we write

$$H^0 = \sum_{\tau=+,-} H^0_{\mathrm{c},\tau} + \sum_{\tau=+,-} H^0_{\mathrm{s},\tau}. \tag{1b}$$

$H^0_{\mathrm{c},\tau}$ is the charge Hamiltonian and $H^0_{\mathrm{s},\tau}$ is the spinon Hamiltonian in the chain $\tau = +, -$. Formally such a decomposition is achieved when we start with a repulsive interaction away from the half-filled band. We replace the fermion fields $\psi_{\sigma,\tau}(x)$ and $\psi^+_{\sigma,\tau}(x)$ by four boson fields $\phi_{\sigma,\tau}(x)$ and the conjugate momentum $\Pi_{\sigma,\tau}(x)$, $[\phi_{\sigma,\tau}(x), \Pi_{\sigma',\tau'}(x')] = \mathrm{i}\delta(x - x')\delta_{\sigma,\sigma'}\delta_{\tau,\tau'}$. The relation between the fermions and the bosons is given by

$$\psi_{\sigma,\tau}(x) = \exp(\mathrm{i}K^{(\tau)}_{\mathrm{F}}x)\,\tilde{\psi}_{\mathrm{L},\sigma,\tau}(x) + \exp(-\mathrm{i}K^{(\tau)}_{\mathrm{F}}x)\,\tilde{\psi}_{\mathrm{R},\sigma,\tau}(x) \tag{2a}$$

$$\tilde{\psi}_{\mathrm{R(L)},\sigma,\tau}(x) = \lim_{\epsilon\to 0} \frac{1}{\sqrt{2\pi\epsilon}} \exp\left[-\mathrm{i}\pi\left(\int_{-\infty}^{x} \mathrm{d}x'\,\Pi_{\sigma,\tau}(x) \pm \phi_{\sigma,\tau}(x)\right)\right]. \tag{2b}$$

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$K_F^{(\tau)}$ is the Fermi momentum for each chain when the number of particles is not fixed; $\phi_{\sigma,\tau}$ and $\Pi_{\sigma,\tau}$ have to be shifted by the integers $N_{\sigma,\tau}$ and $J_{\sigma,\tau}$ [2].

In the case when we are dealing with a ring of length L which is treated by a flux Φ $(\Phi_0 = hc/e) \cdot \theta \equiv \Phi/\Phi_0)$, the field $\psi_{\sigma,\tau}$ is changed to

$$\psi_{\sigma,\tau}(x) \to \exp[\mathrm{i}(2\pi x/L)\theta_x]\,\psi_{\sigma,\tau}(x). \tag{2c}$$

Using equations (2*a*) and (2*b*) we find that the charge in the number of particles and flux gives rise to a trivial shift (zero modes) in $\phi_{\sigma,\tau}$ and $\Pi_{\sigma,\tau}$:

$$\phi_{\sigma,\tau}^{(x)} \to \phi_{\sigma,\tau}(x) + N_{\sigma,\tau}\pi x/L \qquad \Pi_{\sigma,\tau}(x) \to \Pi_{\sigma,\tau}(x) + J_{\sigma,\tau}/L + 2\theta/L \tag{2d}$$

First we ignore these shifts. Then we shall shift our results according to equation (2*d*) when we wish to consider thermodynamics or persistent currents.

Using the representations given by equations (2*a*) and (2*b*) allows us to express the Hamiltonian H^0 given in equation (1*b*):

$$H_{\mathrm{c},\tau}^0 = \int_0^L \mathrm{d}x \left[\tfrac{1}{2}\left(1 - \frac{g}{\pi}\right)\Pi_{\mathrm{c},\tau}^2 + \tfrac{1}{2}\left(1 + \frac{g}{\pi}\right)(\partial_x\phi_{\mathrm{c},\tau})^2\right] \tag{3a}$$

$$H_{\mathrm{s},\tau}^0 = \int_0^L \mathrm{d}x \left[\tfrac{1}{2}\Pi_{\mathrm{s},\tau}^2 + \tfrac{1}{2}(\partial_x\phi_{\mathrm{s},\tau})^2\right]. \tag{3b}$$

Equations (3*a*) and (3*b*) are the usual Luttinger liquid representations given in terms of a repulsive interaction of strength g. Microscopically, one has additional terms; which produce, away from half-filling, a normalization of the parameters $1 - g/\pi$ and $1 + g/\pi$. The relations between $\phi_{\mathrm{c},\tau}$ and $\phi_{\mathrm{s},\tau}$ and the original bosons $\phi_{\uparrow,\tau}$ and $\phi_{\downarrow,\tau}$ are given by

$$\phi_{\mathrm{c},\tau} = (\phi_{\uparrow,\tau} + \phi_{\downarrow,\tau})/\sqrt{2} \qquad \phi_{\mathrm{s},\tau} = (\phi_{\uparrow,\tau} - \phi_{\downarrow,\tau})/\sqrt{2}. \tag{3c}$$

The interaction between the two chains is given by H_{int}:

$$H_{\mathrm{int}} = -J\int_0^L \mathrm{d}x \left(\sum_{\sigma=\uparrow,\downarrow}\sum_{\tau=+,-} :\rho_{\sigma,\tau}(x)\rho_{-\sigma,-\tau}\rho(x):\right) \tag{3d}$$

where J is the coupling between the chains:

$$\rho_{\sigma,\tau}(x) =:_{\epsilon\to 0} \psi_{\sigma,\tau}^{+}(x)\psi_{\sigma,\tau}(x+\epsilon): \tag{3e}$$

Using equations (2*a*), (2*b*) and (3*e*) we replace the interacting Hamiltonian equation (3*d*) by

$$\begin{aligned} H_{\mathrm{int}} &= -J\int_0^L \mathrm{d}x \left(\sum_{\sigma=\uparrow,\downarrow}\sum_{\tau=+,-} :\rho_{\sigma,\tau}(x)\rho_{-\sigma,-\tau}:\right) \\ &= -J\int_0^L \mathrm{d}x \left(\sum_{\sigma=\uparrow,\downarrow}\sum_{\tau=+,-} \frac{1}{\pi}(\partial_x\phi_{\sigma,\tau})(\partial_x\phi_{-\sigma,-\tau}) \right. \\ &\qquad \left. + \frac{1}{2(\pi\epsilon)^2}\cos[2\sqrt{\pi}(\phi_{\sigma,\tau} - \phi_{-\sigma,-\tau}) + 2\,\Delta K_{\mathrm{F}}x]\right) \end{aligned} \tag{3f}$$

where $K_{\mathrm{F}}^{\tau=+} - K_{\mathrm{F}}^{\tau=-} = \Delta K_{\mathrm{F}}$ is the difference between the K_{F} momenta of the two chains. Equations (3*a*), (3*b*) and (3*f*) represent the bosonization representation for the two coupled Luttinger chains.

In the next step we perform an orthogonal transformation and introduce four boson fields: $\phi_{\mathrm{c}}^{(1)}$ carries charge 1e and no spin; $\phi_{\mathrm{c}}^{(2)}$ carries charge 2e and no spin; $\phi_{\mathrm{s}}^{(0)}$ carries no charge and corresponds to spin $S_z = 0, S = 1$; $\phi_{\mathrm{s}}^{(1)}$ carries no charge and spin 1 in the spin $S_z = 1$, $S = 1$ configuration:

$$
\begin{aligned}
\phi_{\mathrm{c}}^{(1)} &= (1/\sqrt{2})(\phi_{\mathrm{c},+} - \phi_{\mathrm{c},-}) = (1/\sqrt{4})(\phi_{\uparrow,+} + \phi_{\downarrow,+} - \phi_{\uparrow,-} - \phi_{\downarrow,-}) \\
\phi_{\mathrm{c}}^{(2)} &= (1/\sqrt{2})(\phi_{\mathrm{c},+} + \phi_{\mathrm{c},-}) = (1/\sqrt{4})(\phi_{\downarrow,+} + \phi_{\downarrow,+} + \phi_{\uparrow,-} + \phi_{\downarrow,-}) \\
\phi_{\mathrm{s}}^{(0)} &= (1/\sqrt{2})(\phi_{\mathrm{s},+} - \phi_{\mathrm{s},-}) = (1/\sqrt{4})(\phi_{\uparrow,+} - \phi_{\downarrow,+} - \phi_{\uparrow,-} + \phi_{\downarrow,-}) \\
\phi_{\mathrm{s}}^{(1)} &= (1/\sqrt{2})(\phi_{\mathrm{s},+} + \phi_{\mathrm{s},-}) = (1/\sqrt{4})(\phi_{\uparrow,+} - \phi_{\downarrow,+} + \phi_{\uparrow,-} - \phi_{\downarrow,-}).
\end{aligned}
\tag{4a}
$$

Using the orthogonal transformation given in equation (4*a*) we replace equations (3*a*), (3*b*) and (3*f*) by the following Hamiltonian:

$$
H = H_{\mathrm{c,s}}^{(1,1)} + H_{\mathrm{s}}^{(0)} + H_{\mathrm{c}}^{(2)}. \tag{4b}
$$

$H_{\mathrm{c,s}}^{(1,1)}$ is represented in terms of the charge-1e *holon* field $\phi_{\mathrm{c}}^{(1)}$ and the *spinon* field $\phi_{\mathrm{s}}^{(1)}$:

$$
\begin{aligned}
H_{\mathrm{c,s}}^{(1,1)} = {} & \tfrac{1}{2}(1-g/2\pi)(\Pi_{\mathrm{c}}^{(1)})^2 + \tfrac{1}{2}(1+g/2\pi - J/\pi)(\partial_x\phi_{\mathrm{c}}^{(1)})^2 + \tfrac{1}{2}(\Pi_{\mathrm{s}}^{(1)})^2 + \tfrac{1}{2}(1+J/\pi)(\partial_x\phi_{\mathrm{s}}^{(1)})^2 \\
& - [2J/(\pi\epsilon)^2]\cos(\sqrt{4\pi}\phi_{\mathrm{c}}^{(1)})\cos(\sqrt{4\pi}\phi_{\mathrm{s}}^{(2)} + 2\Delta\, K_{\mathrm{F}}\, x).
\end{aligned}
\tag{4c}
$$

Because of the chain–chain interaction, equation (4*c*) will develop a gap in the limit $\Delta K_{\mathrm{F}} \to 0$. As a result the charge-1e holon field $\phi_{\mathrm{c}}^{(1)}$ and spinon field $\phi_{\mathrm{s}}^{(1)}$ will have a gap in the excitations:

$$
H_{\mathrm{s}}^{(0)} = \tfrac{1}{2}(\Pi_{\mathrm{s}}^{(0)})^2 + \tfrac{1}{2}(1 - J/\pi)(\partial_x\phi_{\mathrm{s}}^{(0)})^2 \tag{4d}
$$

$$
H_{\mathrm{c}}^{(2)} = \tfrac{1}{2}(1 - g/2\pi)(\Pi_{\mathrm{c}}^{(2)})^2 + \tfrac{1}{2}(1 + g/2\pi - J/\pi)(\partial_x\phi_{\mathrm{c}}^{(2)})^2. \tag{4e}
$$

In contrast to $H_{\mathrm{c,s}}^{(1,1)}$ which has 'massive' excitations, $H_{\mathrm{s}}^{(0)}$ and $H_{\mathrm{c}}^{(2)}$ have 'massless' excitations. This means that there is no gap in the excitation spectrum of the charge-2e *holon* field $\phi_{\mathrm{c}}^{(2)}$ and for the boson field $\phi_{\mathrm{s}}^{(0)}$ in the spin configuration $S_z = 0$, $S = 1$.

This result means that the interaction between the charges make it unfavourable for the holon charge 1e to travel free. In order to do so, one must lose an energy of the order 'J' or break the pair characterized by the holon field $\phi_{\mathrm{c}}^{(2)}$ with charge 2e. The fact that $\phi_{\mathrm{c}}^{(2)}$ and $\phi_{\mathrm{s}}^{(0)}$ (the spin excitation in the triplet configuration $S_z = 0, S = 1$) are massless suggests that the triplet state $S_z = 0, S = 1$, with angular momentum $l = 0$ and isospin $\tau = 0$ is possible. (For each chain we have $\tau = \frac{1}{2}$, $\tau_2 = \pm\frac{1}{2}$, representing localization on one chain or the other.) We would like to mention that such a pairing mechanism is consistent with recent suggestions in the high-T_{c} superconductors [3].

The NMR data (the absence of a Hebel–Shlichter peak which is suggestive of spin triplet pairing) and non-coherent motion in the c direction (the direction perpendicular to the planes) suggest that one can have a pair wavefunction which is s wave like within each plane, a triplet in spin space and antisymmetric with respect to the interchange of the two layers. At this point we wish to show that our situation is in some sense different. We have two chains (no planes) which are a Luttinger liquid (and not a Fermi liquid) for which charge and spin separation takes place. In both cases the matrix elements for single-electron transfer between chains and planes is neglected. In our case we can justify this neglect because of the results obtained in [1]. It was shown by Anderson [4] that, if we take a 1D Luttinger liquid and add single-particle hopping elements which are not too large, the Luttinger liquid is *stable*. After this general remark we can return to our model which is summarized in the results obtained in equations (4*a*)–(4*d*). We shall use these results to describe the pairing order parameter $\Delta(x)$ in terms of the four boson fields $\phi_{\mathrm{c}}^{(1)}$, $\phi_{\mathrm{c}}^{(2)}$, ϕ_{s}^{0} and $\phi_{\mathrm{s}}^{(1)}$. We consider the triplet pairing in the state $s = 1$, $s_z = 0$, $\tau = 0$:

$$
\Delta(x) = [\tilde{\psi}_{\mathrm{R},\uparrow,+}^{+}(x)\tilde{\psi}_{\mathrm{L},\downarrow,-}^{+} - \tilde{\psi}_{\mathrm{L},\uparrow,-}^{+}(x)\tilde{\psi}_{\mathrm{R},\downarrow,+}^{+}] + [\tilde{\psi}_{\mathrm{R},\uparrow,-}^{+}(x)\tilde{\psi}_{\mathrm{L},\downarrow,+}^{+}(x) - \tilde{\psi}_{\mathrm{L},\downarrow,+}^{+}(x)\tilde{\psi}_{\mathrm{R},\downarrow,-}^{+}(x)]. \tag{5a}
$$

Using bosonic language we obtain

$$\begin{aligned}\Delta(x) = \Big[&\exp\Big(\mathrm{i}\sqrt{\pi}(\phi_{\uparrow,+} - \phi_{\downarrow,-}) + \mathrm{i}\sqrt{\pi}\int_{-\infty}^{x} \mathrm{d}x' \,[\Pi_{\uparrow,+}(x') + \Pi_{\downarrow,-}(x')]\Big) \\ &- \exp\Big(-\mathrm{i}\sqrt{\pi}(\phi_{\uparrow,-} - \phi_{\downarrow,+}) + \mathrm{i}\sqrt{\pi}\int_{-\infty}^{L} \mathrm{d}x' (\Pi_{\uparrow,-} + \Pi_{\downarrow,+})\Big)\Big] \\ &+ \Big[\exp\Big(\mathrm{i}\sqrt{\pi}(\phi_{\uparrow,-} - \phi_{\downarrow,+}) + \mathrm{i}\sqrt{\pi}\int_{-\infty}^{x} \mathrm{d}x' \,[\Pi_{\uparrow,-}(x') + \Pi_{\downarrow,+}(x')]\Big) \\ &- \exp\Big(-\mathrm{i}\sqrt{\pi}(\phi_{\uparrow,+} - \phi_{\downarrow,-}) + \mathrm{i}\sqrt{\pi}\int_{-\infty}^{x} \mathrm{d}x' \,[\Pi_{\uparrow,+}(x') + \Pi_{\downarrow,-}(x')]\Big)\Big] \\ = \Big[&\exp\Big(\mathrm{i}\sqrt{\phi}(\phi_c^{(1)} + \pi_s^{(1)}) + \mathrm{i}\sqrt{\pi}\int_{-\infty}^{x} \mathrm{d}x' \,[\Pi_c^{(2)}(x') + \Pi_s^{(0)}(x')]\Big) \\ &- \exp\Big(\mathrm{i}\sqrt{\pi}(\phi_c^{(1)} - \phi_s^{(1)}) + \mathrm{i}\sqrt{\pi}\int_{-\infty}^{x} \mathrm{d}x' \,[\Pi_c^{(2)}(x') - \Pi_s^{(0)}(x')]\Big)\Big] \\ &+ \Big[\exp\Big(-\mathrm{i}\sqrt{\pi}(\phi_c^{(1)} - \phi_s^{(1)}) + \mathrm{i}\sqrt{\pi}\int_{-\infty}^{x} \mathrm{d}x' \,[\Pi_c^{(2)}(x') - \Pi_s^{(0)}(x')]\Big) \\ &- \exp\Big(-\mathrm{i}\sqrt{\pi}(\phi_c^{(1)} + \phi_s^{(1)}) + \mathrm{i}\sqrt{\pi}\int_{-\infty}^{x} \mathrm{d}x' \,[\Pi_c^{(2)}(x') + \Pi_s^{(0)}(x')]\Big)\Big]. \end{aligned} \tag{5b}$$

We then employ the fact that the fields $\phi_c^{(2)}$, $\Pi_c^{(2)}$ and $\phi_s^{(0)}$, $\Pi_s^{(0)}$ are massless; using the representation given in equation (5*b*) we compute the pair–pair correlation for the triplet state and $\tau = 0$. We find that

$$\begin{aligned}\langle\Delta(x,t)\Delta^*(x+R,t')\rangle \simeq &[(R - V_c^{(2)}|t-t'| + \mathrm{i}/\epsilon)(R + V_c^{(2)}|t-t'| - \mathrm{i}/\epsilon)]^{-\beta_c^{(2)}} \\ &\times [(R - V_s^{(0)}|t-t'| + \mathrm{i}/\epsilon)(R + V_s^{(0)}|t-t'| - \mathrm{i}/\epsilon)]^{-\beta_s^{(0)}}\end{aligned} \tag{5c}$$

where $V_c^{(2)}$, $V_s^{(0)}$ and $\beta_c^{(2)}$, $\beta_s^{(0)}$ are the *velocities* and the *correlation exponents* of the two Luttinger liquids given in equations (4*e*) and (4*d*):

$$\begin{aligned} &V_c^{(2)} = \sqrt{(1 + g/2\pi - J/\pi)/(1 - g/2\pi)} \qquad V_s^{(0)} = \sqrt{1 - J/\pi} \\ &\beta_c^{(2)} \sim 1/4\pi\sqrt{1 - g/2\pi} \qquad \beta_s^{(0)} \sim 1/4\pi. \end{aligned} \tag{5d}$$

Finally, we would like to comment on the *persistent* current of two *coupled rings* treated by the flux Φ. According to equations (2*c*) and (2*d*) the effect of the flux is to shift $\Pi_{\sigma,\tau}(x)$ by $2\theta/L$, $\theta = \Phi/\Phi_0$.

Owing to the coupling between the rings we perform the orthogonal transformation given in equation (4*a*). As a result we find that the shift in the charge-$2e$ holon is

$$\Pi_c^{(2)} \to \Pi_c^{(2)} + 4\theta/L \tag{6a}$$

which is the same as a shift in $2\theta'/L$ with $\theta' = \Phi/\frac{1}{2}\Phi_0$. As a result the persistent current will have a period of $\frac{1}{2}\Phi_0 = hc/2e$ and not Φ_0.

To conclude we have shown that the coupling between two Luttinger chains produces a gapless excitation for charge-$2e$ bosons.

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